# Pinhole Optics

Science, at bottom, is really anti-intellectual. It always distrusts pure reason and demands the production of the objective fact.

H. L. Mencken (1880-1956)

## OBJECTIVES

To study the formation of an image without use of a lens.

## THEORY

A pinhole camera consists of a darkened box or room with a small hole at one end. Because light travels in straight lines, the hole permits rays from each point of an object to fall only within a small circle on the opposite wall, effectively forming an image. As the pinhole is made smaller the image will become more distinct until the hole is so small that diffraction becomes important.

Although pinhole cameras were probably known to the ancient Greeks, they are still used in preference to lens systems in some situations. Pinholes are obviously useful for imaging x-rays or particle streams, where no lens materials are available, but even for light they offer complete freedom from linear distortion, virtually infinite depth of focus and a very wide angular field. Modest resolution and a very dim image are the disadvantages. Overall, pinhole cameras are worth study because they are useful and also because they illustrate some interesting physics.

In this exercise we will examine the angular resolution of the pinhole camera as a function of hole size, as we might do to design a camera for a particular use. The optimum pinhole size must be a compromise between the large spot produced by a large hole and the large spot produced by diffraction from a very small hole. It is fairly easy to calculate the expected resolution for large holes, which we will call the geometric limit, and for small holes, the diffraction limit, but in between the situation is more complicated so we will resort to an empirical study to locate the conditions for the best resolution.

As shown in Fig. 1, which defines our notation, a point on the object casts a shadow of the hole onto the wall of the camera. For a large hole of diameter $a$ the radius of the shadow, $g$, is given by

$$g = \frac{a}{2}\frac{p+q}{p} \tag{1}$$

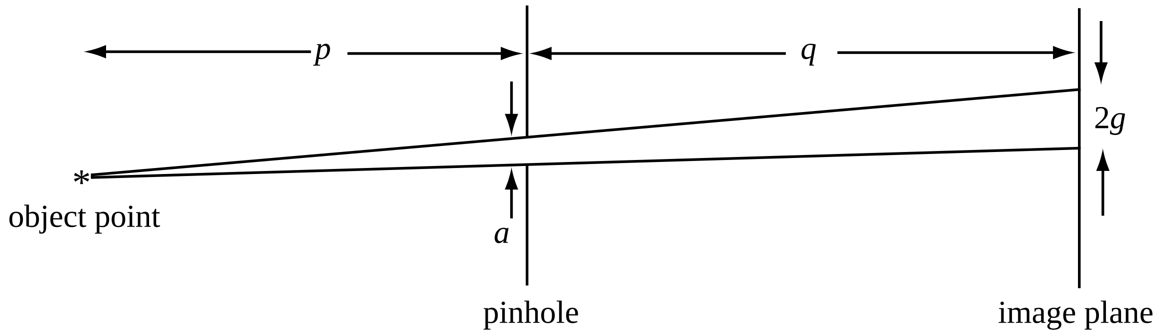
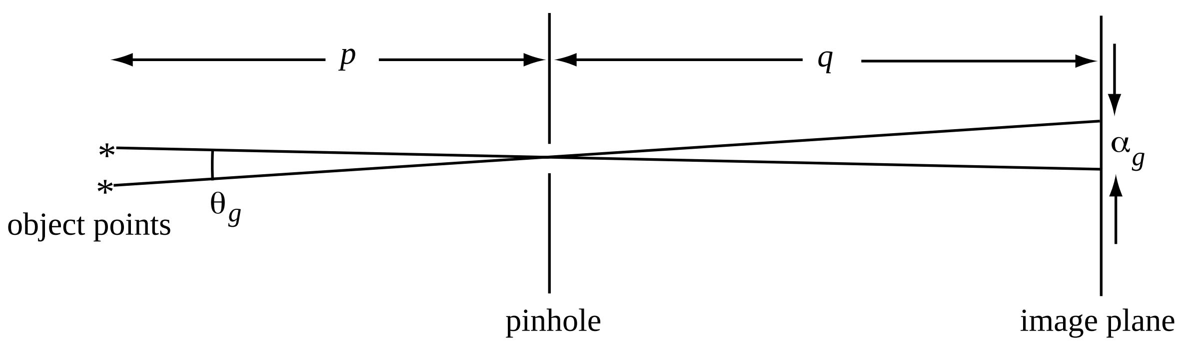



Fig. 1 Schematic of pinhole camera, showing how a point object a distance $p$ in front of a pinhole of diameter $a$ produces an image of diameter $2g$ at the image plane a distance $q$ behind the pinhole.

Two different points on the object will cast two shadows, as indicated in Fig. 2. We will be able to tell that there are two distinct points when the centers of the shadows are sufficiently separated, say by a distance $\alpha g$, where $\alpha$ is about 2. Calling the angular separation of two points that are just resolved $\theta_g$, we find

$$\theta_g \approx \tan\theta_g = \frac{\alpha g}{q} = \frac{\alpha a}{2}\frac{p+q}{pq} \tag{2}$$

When the hole is quite small, the image of a point source is not a single disk, but a bright region surrounded by concentric light and dark diffraction rings, with the size of the central region inversely proportional to the hole diameter. Two such patterns are generally considered to be resolved when the central disk of each pattern just overlaps the first dark ring of the other pattern. Using this criterion, we find that two points are resolved in the diffraction-limited regime



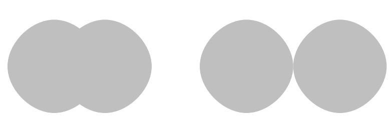



Fig. 2 The top illustration shows two points on the object forming circular images centered a distance $\alpha g$ apart in the image plane. Below are shown typical pairs for $\alpha = 1$, definitely not resolved, and $\alpha = 2$, which is well resolved.

when their angular separation is

$$\theta_d = 1.22\frac{\lambda}{a} \tag{3}$$

where $\lambda$ is the wavelength of light used.

A qualitative sketch of Eq. 2 and 3, as in Fig. 3, suggests that there is an optimum value of $a$ for any chosen $p$, $q$ and $\lambda$. It would be tedious to find the optimum experimentally for a group of parameters, but fortunately the problem can be simplified by using scaled variables. Specifically, let

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \tag{4}$$

by analogy with the usual lens formula so that

$$\theta_g = \frac{\alpha a}{2f} \tag{5}$$

Define dimensionless scaled quantities $a'$ and $\theta'$ by

$$a' = a\big/\sqrt{\lambda f} \qquad \theta' = \theta\big/\sqrt{\lambda / f} \tag{6}$$

to arrive at

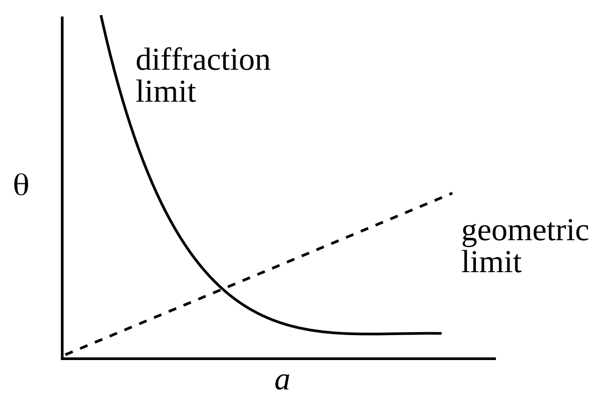



Fig. 3 Sketch of the angular resolution as a function of pinhole diameter in the diffraction limit (solid) and geometric limit (dashed). Neither description is expected to apply in the crossover region.

$$\theta'_g = \frac{\alpha}{2} a' \qquad \theta'_d = \frac{1.22}{a'} \tag{7}$$

This shows that a plot of q' vs a' will be universal. We can make measurements for convenient values of a, p, q, and then calculate the resolution to be expected for any desired parameters within the range of our scaled variables.

The last item of theoretical business is to define what we mean by 'large' and 'small' holes. The geometric and diffraction-limit lines cross at $a' = 2.44/\alpha$, near $a' = 1$ in Fig. 3. We therefore expect the geometric limit to hold when $a' >> 1$ or equivalently $a^2/\lambda >> f$, and the diffraction limit when $a^2/\lambda << f$. Around $a' = 1$ we do not expect either approximation to be accurate, as we will demonstrate from measurements.

**EXPERIMENTAL PROCEDURE**

Figure 4 shows the general layout of the components. The target and camera box are on an optical bench, so that the distance $p$ can be adjusted easily. The pinholes are in pieces of photographic film which mount magnetically on the camera box. Center one of the pinholes over the opening in the side of the box, removing the box lid to facilitate alignment if necessary. Place the lamp near the end of the optical bench so it illuminates the target, but keep it at least 25 cm from the target to avoid heat damage. When the lamp is turned on you should see a dim image of the target on the screen at the back of the camera box.

The resolution target consists of groups of three black bars, with each group having systematically smaller spacing. The resolution limit is the separation of the group that can just be distinguished as three bars, rather than simply a gray blob. Although subjective, the transition from resolved to unresolved is usually fairly clear and reproducible between observers. Using a magnifier will help when the image is small. It is also important to look for three bars, so that you do not become confused by spurious resolution, a phenomenon explained in Fig. 5.

To get a reasonable range of scaled hole diameters you should determine which bar pattern is just resolved for each of the eight hole sizes for $p$ = 15 and $p$ = 50 cm. Using the camera specifications in Table 1 and the bar spacings in Table 2 you can convert your measurements to scaled variables $a'$ and $\theta'$, and then plot $\theta'$ as a function of $a'$. Plot the

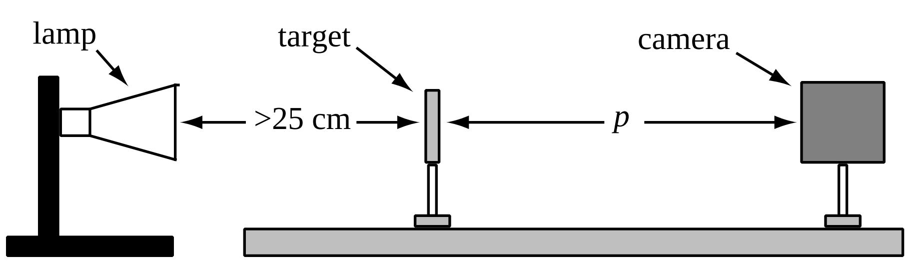
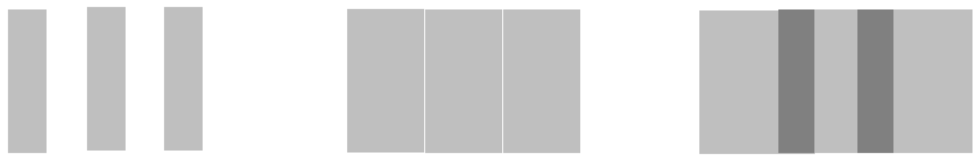



Fig. 4 Overall layout of the apparatus.



(a) (b) (c)

Fig. 5 Resolution-limited images of three uniform bars. In (a) the bars are clearly separate and resolved. In (b) the bars have blurred and are not resolved. In (c) further blurring produces overlap and the appearance of two resolved images. This effect can occur with any periodic structure, and may be misinterpreted as a real property of the object being viewed.

theoretical limits from Eq. 7 on the same graph, fitting a straight line to your data to determine $\alpha$. Since we are not imaging in monochromatic light, you can only approximate the wavelength as $\lambda$ = 550 nm, the mean for the visible band.

**REPORT**

In your report you should discuss the form of the $a'$, $\theta'$ graph, with particular attention to the region near crossover. Also, provide a geometrical interpretation of the value of $\alpha$ that fits your data for large hole sizes.

Table 1 Camera Specifications

Image distance, $q$ = 10.1 cm

Pinhole diameters, slide 1, $a$ = 0.11 mm, 0.22 mm, 0.30 mm, 0.40 mm

Pinhole diameters, slide 2, $a$ = 0.59 mm, 0.78 mm, 0.98 mm, 1.17 mm

Table 2 Resolution Chart Bar Spacings (mm)

| | -2 | -1 | 0 | 1 |
|---|---|---|---|---|
| 1 | 5.47 | 2.71 | 1.39 | 0.69 |
| 2 | 4.87 | 2.40 | 1.20 | 0.60 |
| 3 | 4.31 | 2.17 | 1.07 | 0.55 |
| 4 | 3.88 | 1.89 | 0.96 | 0.47 |
| 5 | 3.47 | 1.70 | 0.86 | 0.41 |
| 6 | 3.02 | 1.47 | 0.75 | 0.36 |